

Fuzzy-based Parameterized Gaussian Edge Detector Using Global and Local Properties

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Abstract

Many edge detection schemes suffer from the lack of image quality at the global level. Global properties are more vital in grayscale images due to loss of hue and texture. This paper proposes a novel fuzzy-based Gaussian edge detector that uses both global and local image properties for grayscale images. In the global contrast intensification phase, each pixel in an image is represented in the fuzzy domain using a modified Gaussian membership function. A nonlinear contrast intensification function containing three parameters is used to further enhance the image. In the local phase, we present a novel fuzzy parameterized Gaussian-type edge detector mask containing two fuzzifier parameters, which are chosen based on experimental selection rules. Optionally, the fuzzy image entropy function can be used to optimize all the parameters through simple gradient descent technique. In experiments conducted on various classic images, this algorithm showed notable visual improvement on both strong and weak edges in comparison with common edge detectors.

Keywords – Image enhancement, Gaussian edge detector, Gaussian membership function, contrast intensification, entropy optimization.

1. Introduction

In computer vision and pattern recognition, edge detection is a useful low-level image processing tool for image analysis and interpretation, as well as a segmentation tool for various recognition applications. Edges and contours are often useful features as they represent an image by its object boundaries and separation of dissimilar regions in terms of pixel intensities. Furthermore, edges are considered important elements in a picture, as they present essential information of an object of interest in a picture.

The basic mechanism of most edge detection techniques is to devise a local first or second derivative

operator, followed by some regularization technique to reduce the effects of noise. Early edge detection methods, such as the Sobel and Prewitt detectors [1], [2] were based on the concept of spatial derivative filtering, where local gradient operators are used to detect edges of certain orientations only. Derivative filters suffer when the edges are blurred and noisy and are not flexible.

Gaussian filter is biologically motivated by the mechanisms of the human visual system and it was shown by Marr and Hildreth [3] that this filter (along with the Laplacian operator) is a very close approximation to the shape of spatial receptive fields in the visual system of cats. They proposed an algorithm that finds edges at the zero-crossings of the Laplacian of an image. Canny [4] proposed a method to counter noise problems from gradient operators, where the image is convolved with the first-order derivatives of Gaussian filter for smoothing in the local gradient direction followed by edge detection by thresholding. Non-linear techniques such as the SUSAN edge detector [5] have also gained much popularity.

In recent years, fuzzy-based edge detection techniques have also shown much promise in the areas of computer vision and image processing. Fuzzy techniques offer a new perspective to discern the imperfections of image processing. Many recent techniques have characterized edge detection as a fuzzy reasoning problem. Bloch [6] used fuzzy sets as the basis of a morphological edge extraction method while Russo and Ramponi [7] utilized fuzzy rule-based operators built on IF-THEN-ELSE rule-based architecture for edge detection. In both the cases, global information was evidently neglected. Later on, Ho and Ohnishi [8] proposed a fuzzy edge detector that uses both global and local image information for fuzzy categorization and classification (FCC) based on edges.

In view of the above strengths, we have proposed a fuzzy-based parameterized Gaussian approach to edge detection that utilizes both global and local image information. Firstly, a modified Gaussian membership function is used to represent each pixel in the fuzzy domain. Then, a global nonlinear contrast intensification operator acts to enhance the edginess of image pixels and

flatten obscure textures. The actual edge detection takes place in the local phase, where a novel fuzzy Gaussian-type edge detector mask is invoked. An improved selection of parameter values can be achieved through fuzzy entropy optimization.

The remainder of this paper is organized as follows: Sections 2 and 3 introduce the global contrast intensification phase and local edge detector phase respectively, while fuzzy entropy optimization is elaborated in Section 4. Section 5 briefly sums up the proposed algorithm. Section 6 discusses the experimental results, analysis and future directions. Lastly, Section 7 provides conclusions of this paper.

2. Global Contrast Intensification

2.1 Fuzzy image representation

First, to represent a spatial domain image in the fuzzy domain, a gray tone image X of dimension $M \times N$, with L gray levels, is considered as a 2D array of fuzzy singleton sets,

$$X = \{(x_{mn}, \mu_{mn}(x)) | (m, n) \in [(0, 0), (M-1, N-1)]\}, \quad (1)$$

where each pixel is characterized by the intensity value x_{mn} and its grade of possessing some membership μ_{mn} ($0 \leq \mu_{mn} \leq 1$). Each membership value corresponds to the spatial intensity value of the image in the range $[0, L-1]$.

2.2 Histogram-based fuzzy membership function

Fuzzy property can be expressed in terms of a membership function. For the transformation of a grayscale image X in the range of $[0, L-1]$ to the fuzzy property plane in the interval $[0, 1]$, a Gaussian membership function of the form

$$\mu(x_{mn}) = e^{[-(x_{\max} - x_{mn})^2 / 2f_h^2]} \quad (2)$$

was suggested in [9], and contains a single fuzzifier, f_h . x_{\max} and x_{mn} are the maximum and (m, n) th gray values respectively.

Here, we propose a histogram-based membership function to represent pixels of the spatial domain in the fuzzy domain by

$$\mu(k) = e^{[-(x_{\max} - k)^2 / 2f_h^2]} \quad (3)$$

where k is a certain gray value in the range $[0, L-1]$, and the fuzzifier parameter, f_h can be determined as

$$f_h^2 = \frac{\sum_{k=0}^{L-1} (x_{\max} - k)^4 p(k)}{2 \sum_{k=0}^{L-1} (x_{\max} - k)^2 p(k)} \quad (4)$$

where $p(k)$ stands for the frequency of occurrence of intensity level k in histogram H_X , which is

$$p(k) = \frac{H_X(k)}{(M-1)(N-1)} \quad (5)$$

In the fuzzy plane, pixels of a contrast-enhanced image are either of low perception (dark), *i.e.*, $\mu \in [0, 0.5]$, or of high perception (bright), *i.e.*, $\mu \in [0.5, 1]$ values. This leaves pixels near $\mu = 0.5$ having the highest ambiguity and do not belong to either perception class. Assuming that these pixels describe the fuzzy boundary, values near 0.5 are considered to contain edges.

2.3 Contrast intensification function

Poor contrast of degraded images is usually enhanced by common methods such as histogram equalization stretching and gray level transformations. Zadeh [10] first introduced the contrast intensification operator, $INT[\mu(k)]$, which solely depends on the membership function alone. As image degradation is nonlinear and indeterminate in nature, we use the nonlinear contrast intensification function $NINT[\mu(k)]$ introduced in [14], which contains 3 tunable parameters, *viz.*, intensification operator t and the crossover point x_c , and the fuzzifier f_h from Equation (3). This function is a parametric sigmoid given by

$$\mu'(k) = NINT[\mu(k)] = \frac{1}{1 + \exp[-t(\mu(k) - x_c)]} \quad (6)$$

where t controls the shape of the sigmoid function.

Parameters f_h and x_c are adjusted through $\mu(k)$ while the intensification operator t will be fixed to control the level of contrast enhancement in the image.

3. Local Edge Detection

3.1 Local edge detector mask

By redefining the contrast intensification function, $NINT[\mu(k)]$ in terms of every (m, n) th pixel,

$$\mu'(m, n) = NINT[\mu_{mn}] = \frac{1}{1 + \exp[-t(\mu_{mn} - x_c)]} \quad (7)$$

we propose a novel fuzzy parameterized Gaussian-type edge detector given as

$$\eta(m, n) = e^{-\sum_i \sum_j \left(\frac{[\mu'(m+i, n+j) - \mu'(m, n)]}{f_g} \right)^\alpha} \quad (8)$$

where $i, j \in [-(w-1)/2, (w-1)/2]$, and $w \times w$ is the size of the edge detector mask. $\mu'(m, n)$ is the membership value of central pixel of the mask at location (m, n) and $\eta(m, n)$ is the output edge pixel replacing the previous central pixel. Parameters α and f_g are both adjustable fuzzifiers of the Gaussian mask function.

Theoretically, parameters α and f_g represent the Gaussian function more meaningfully than the previously

proposed parameters (α and β) in [11]. With a general Gaussian membership function,

$$\mu_{GAUSSIAN}(x; \sigma, c) = \exp\left[\frac{-(x-c)^2}{2\sigma^2}\right] \quad (9)$$

the Gaussian curve depends on parameters σ and c . While the parameter α is taken as the power of the exponential term, the denominator of the exponential term is reduced to a single fuzzifier constant, f_g as in

$$\mu_{GAUSSIAN} = \exp\left[\left(\frac{-(x-c)}{2\sigma}\right)^\alpha\right] \\ \Rightarrow 2\sigma \equiv f_g \quad (10)$$

3.2 Selection of fuzzifiers

As the mask is a general Gaussian-type function, different values of α and f_g would yield different variations of the Gaussian function. By experimenting with different values of fuzzifier α , the mask performance is found to be visually poor with lumpy and thick edges when α takes an even integer while odd integers of α will result in thinner and refined edges. Moreover low odd values tend to yield too many weak edges and conversely, many weak edges are eliminated on high odd values.

In mathematical terms, the fuzzifier f_g defines the width of the Gaussian function. Here, the selection of a low f_g value would produce a narrower Gaussian mask, which in turn reduces the approximation distance to a crisp set. The closer the function to a crisp set, the more is edge information, and vice versa for high f_g values.

To ensure good performance in a required application, both parameters α and f_g should be pre-determined through initial experimentation and entropy optimization (see Section 4).

3.3 Removal of strong edges and impulse noise

Strong edge and impulse noise are encountered when we have

- large values of η , i.e. $\eta(m,n) > 1$; and
- small values of η , i.e. $\eta(m,n) < 0$

By applying the following *edge attenuator* scheme, we can contain the edge values within an approximate range of [0,1]:

$$\eta'(m,n) = \min[\{\eta(m,n) \mid m \in [0, M-1], n \in [0, N-1]\}], \\ \forall \eta(m,n) > 1; \\ \text{AND} \\ \eta'(m,n) = \max[\{\eta(m,n) \mid m \in [0, M-1], n \in [0, N-1]\}], \\ \forall \eta(m,n) < 0 \quad (11)$$

3.4 Edge image thresholding

Finally, a simple thresholding can be applied to produce a binarized edge image. An optimum threshold level λ determined through experiments was found to be in the range of 0.75 to 0.95,

$$\eta''(m,n) = \begin{cases} 1, & \text{for } \eta'(m,n) \geq \lambda \\ 0, & \text{for } \eta'(m,n) < \lambda \end{cases} \quad (12)$$

where $\lambda = 0.75 \rightarrow 0.95$, and $m \in [0, M-1], n \in [0, N-1]$.

Image thresholding is an optional step, and the need for it depends on how relevant it is to an application.

4. Fuzzy Entropy Optimization

We employ fuzzy entropy optimization to estimate four tunable parameters: x_c, f_h, α and f_g . The entropy of a fuzzy set is a measure of the degree of fuzziness of that set and provides the indefiniteness of an image. Entropy E [12], using Shannon's function S_e , is defined by

$$E = \frac{1}{\ln 2} \sum_{k=0}^{L-1} S_e p(k) \quad (13)$$

and in fuzzy terms, we define S_e as

$$S_e(\mu'(k)) = -\mu'(k) \ln \mu'(k) - (1 - \mu'(k)) \ln(1 - \mu'(k)), \\ 0 \leq \mu' \leq 1 \quad (14)$$

4.1 Optimization of x_c and f_h

Among the three tunable parameters in the global phase, only parameters x_c and f_h are optimized using entropy optimization. Optimization is not needed for the predetermined parameter t .

The derivatives of E with respect to x_c and f_h are obtained as:

$$\frac{\partial E}{\partial x_c} = \frac{\partial E}{\partial \mu'(k)} \frac{\partial \mu'(k)}{\partial x_c} \\ = \frac{1}{\ln 2} \sum_{k=0}^{L-1} [t^2(\mu(k) - x_c)g(\mu')]p(k) \quad (15)$$

$$\frac{\partial E}{\partial f_h} = \frac{\partial E}{\partial \mu'(k)} \frac{\partial \mu'(k)}{\partial f_h} \\ = \frac{1}{\ln 2} \sum_{k=0}^{L-1} \left[\frac{t^2 \mu(k)(\mu(k) - x_c)(x_{\max} - k)^2 g(\mu')}{f_h^3} \right] p(k) \quad (16)$$

where $g(\mu')$ is denoted as

$$g(\mu') = \mu'(k)(1 - \mu'(k)) = \frac{e^{-t(\mu(k) - x_c)}}{[1 + e^{-t(\mu(k) - x_c)}]^2} \quad (17)$$

These derivatives are used in the recursive learning of the parameters x_c and f_h by gradient descent technique:

$$x_c' = x_c - \varepsilon_1 \frac{\partial E}{\partial x_c} \quad (18)$$

$$f_h' = f_h - \varepsilon_2 \frac{\partial E}{\partial f_h} \quad (19)$$

where ε_1 and ε_2 are learning rates for both parameters x_c and f_h respectively. The nearest optimization point of both positive and negative search directions is taken.

4.2 Optimization of α and f_g

In the local phase that involves the decisive step of extracting edges, optimization can also be applied to find optimized values for parameters α and f_g . Considering the fuzzy entropy function at the local level, we modify Equation (14) by replacing $\mu'(k)$ with the local edge pixel $\eta(m,n)$,

$$E(\eta(m,n)) = -[\eta(m,n) \ln \eta(m,n) + (1 - \eta(m,n)) \ln(1 - \eta(m,n))] \quad (20)$$

The derivatives of E with respect to α and f_g are obtained as:

$$\begin{aligned} \frac{\partial E}{\partial \alpha} &= \frac{\partial E}{\partial \eta(m,n)} \frac{\partial \eta(m,n)}{\partial \alpha} \\ &= \eta \ln \left\{ \frac{\eta}{1-\eta} \right\} \sum_i \sum_j \left[\left(\frac{K}{f_g} \right)^\alpha \ln \left(\frac{K}{f_g} \right) \right] \end{aligned} \quad (21)$$

$$\begin{aligned} \frac{\partial E}{\partial f_g} &= \frac{\partial E}{\partial \eta(m,n)} \frac{\partial \eta(m,n)}{\partial f_g} \\ &= \frac{\eta \alpha \sum_i \sum_j K^\alpha}{f_g^{\alpha+1}} \ln \left\{ \frac{\eta}{1-\eta} \right\} \end{aligned} \quad (22)$$

where $K = [\mu'(m+i, n+j) - \mu'(x, y)]$.

Using the gradient descent technique, the above derivatives are applied recursively to learn the parameters:

$$\alpha' = \alpha - \varepsilon_3 \frac{\partial E}{\partial \alpha} \quad (23)$$

$$f_g' = f_g - \varepsilon_4 \frac{\partial E}{\partial f_g} \quad (24)$$

where ε_3 and ε_4 are learning factors for both parameters α and f_g respectively. The feasible range of values for α and f_g to avoid the problem of convergence and divergence are as follows:

- $\alpha - 0.25 \leq \alpha' \leq \alpha + 0.25$
- $30 \leq f_g' \leq 100$

If the values of α and f_g exceeds the range specified, optimization is halted, and the old values are retained.

5. Algorithm Flow

The algorithm flow of the fuzzy-based Gaussian edge detection algorithm is as follows:

Global Phase:

1. Determine value of the parameter f_h .
2. Determine $\mu(k)$ for $k=1, \dots, L-1$.
3. Determine $\mu'(k)$ using NINT $[\mu(k)]$.
4. Optimize x_c and f_h . (*Optional*)
 - i. Repeat step 2 with new f_h .
 - ii. Repeat step 3 with new x_c .

Local Phase:

5. Represent the membership $\mu'(k)$ in pixel level as $\mu'(m,n)$.
6. Apply an edge detector mask to obtain the edge output, $\eta(m,n)$.
7. Optimize α and f_g . (*Optional*)
 - i. Repeat step 6 with new α and f_g .
8. Remove strong edges and impulse noise to produce he filtered edge output, $\eta'(m,n)$.
9. Binarize the edge output at a threshold level, λ to obtain binary edge output. (*Optional*)

6. Results and Discussions

In our experiments, the proposed algorithm was implemented on 3 classic test images – the ‘Lena’, ‘Cameraman’ and ‘Barbara’ images; all grayscale images of size 256x256. Prior to the experiments, no pre-processing was done on these images.

As the algorithm has two phases – global enhancement phase and local detection phase, we present the experimental results and discussion separately. For clarity of illustration, we use the Lena image mostly in our visual assessment and analysis.

6.1 Experimental results and analysis

I. Global Phase

The contrast intensification operator that performs global enhancement contains 3 parameters, x_c , f_h , t . Parameter t is preset and values of $5 < t < 7$ would show a reasonable amount of contrast boost. Initially, x_c is taken as $x_c = 0.5$. Upon optimization, x_c varies around 0.5, or may reach as high as 0.7 in some cases. The initial value of f_h is obtained from Equation (4) and usually varies within 5-15 units after optimization. The quantitative result of enhancement, before and after entropy optimization, for the three test images is shown in Table 1.

The original and enhanced images of Lena are shown in Fig. 1. After enhancement, the weak edges appear to have sufficient contrast and this allows better discrimination of the weaker edges during edge detection.

Table 1. Entropy and contrast intensification parameter values for the Lena, Cameraman, and Barbara images

Image		E	f_h	t preset	x_c
Stage*					
Lena 256x256	I	0.6719	116.5739	5.00	0.5000
	EU	0.5941	120.5549	5.00	0.5000
	EO	0.6354	122.2986	5.00	0.5783
Cameraman 256x256	I	0.5490	120.2502	5.00	0.5000
	EU	0.5641	96.0800	5.00	0.5000
	EO	0.7800	108.7025	5.00	0.6931
Barbara 256x256	I	0.6596	116.7062	5.00	0.5000
	EU	0.6249	104.3196	5.00	0.5000
	EO	0.7069	110.3554	5.00	0.5974

*I: Initial; EU: Enhanced, unoptimized; EO: Enhanced, optimized



Fig. 1. The original (left) and, enhanced and parameter-optimized (right) Lena image with $t = 5$.

II. Local Phase

In the local phase, the proposed mask contains 2 tunable parameters, α and f_g . α is the measure of edge strength; high α values yield prominent edges but discard weak ones while low α values keep both the prominent edges and most of the weak edges as well. f_g gives the fuzziness or scale of visibility of edges in the image; smaller values of f_g show more textured details or edge information, and vice versa.

By these experiments, suitable values of α are found in the range of 2.8–3.2, centered around $\alpha = 3$. Values above or below this range would produce thick, lumpy edges. The values of f_g range from 30–100, where f_g below 30 would result in much unwanted texture and speckled edges, while f_g above 100 would purge too much edge information. Table 2 shows the initial and new optimized values of α and f_g for the three test images.

The Lena edge images before and after entropy optimization are shown in Fig. 2. By visual assessment, we find that the overall basic object shape is retained in the Lena image, with a good balance of detail and textured edges. In addition to that, the sketchy and jagged edges that were produced with our initial parameters also visibly improved by using the optimized parameters.

Table 2. Initial and new optimized values for parameters α and f_g for the Lena, Cameraman, and Barbara test images. The optimized global enhancement parameters from Table 1 are used for their respective images.

Image (with learning factors)	α	α'	f_g	f_g'
Lena $\epsilon_3 = 0.001, \epsilon_4 = 0.0001$	2.9500	2.9819	63.0000	60.3985
Cameraman $\epsilon_3 = 0.1, \epsilon_4 = 0.1$	2.9000	2.8987	72.0000	73.9379
Barbara $\epsilon_3 = 0.01, \epsilon_4 = 0.2$	3.2000	2.7706	57.0000	89.5218



Fig. 2. Binary edge output image for Lena with initial parameters (left) and optimized parameters (right). Both edge images are thresholded at $\lambda = 0.75$.

III. Comparative Analysis

For comparison, we have applied the Canny [4], and scale-space Gaussian gradient diffusion [13] edge detectors on the Lena test image as well (Fig. 3). All three methods produced very contrasting outcomes. The Canny detector yields many needless edges and does not seem to preserve vital shape details whereas the proposed detector favors many important edges that define the overall shape. The Gaussian gradient diffusion operator causes blurred and thick edges despite having its advantages of edge clarity. Most valuable texture details of the face, hair and hat are lost. Even by human visual perception, we can conclude that the proposed algorithm is able to produce an edge image that has the most recognizable face.

Further comparison with the Canny detector using the Cameraman image is shown in Fig. 4. Note that the shape of the face (in side view), camera tripod and the structure of the background buildings are well maintained and

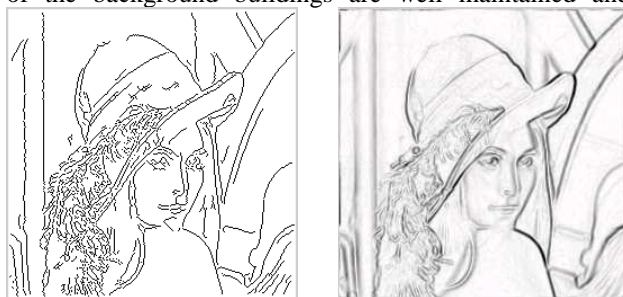


Fig. 3. Canny edge operator (left) and Gaussian gradient diffusion edge operator (right)

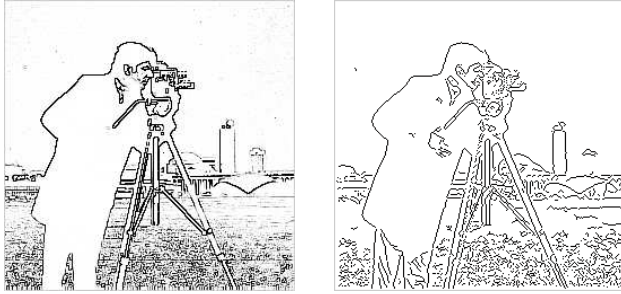


Fig. 4. Edge output of the Cameraman image using the proposed fuzzy edge detector (left) and Canny edge detector (right)

clearly traced out while the Canny edges are messy, resulting in much uncertainty in the shape of objects.

6.2 Discussion and future work

We have made a visual assessment of detected edges for shape to ascertain the edge quality, as done in most past literature.

The enhancement phase is essential for creating contrast in the images as lack of contrast often leads to loss of sharp details and texture. Generally, enhancement is not part of many existing edge detectors in the literature. This is one reason they do not fare well in the shape preservation of objects. Thus, this added feature makes it different from other common edge detectors.

The proposed edge detector is also tailored with the ability to reduce the number of false edges while strengthening significant edges. Unlike the Canny detector, which uses directional gradients, this edge detector is built on the basis of nonlinear functions. Thus, edges can be selected according to their significance and strength depending on the parameter set. This detector has another distinctive feature – it retains the texture of the original image.

Although the performance of the proposed fuzzy edge detector excels as a shape and texture detector, it suffers from two drawbacks that can be easily mitigated. In our future work, these drawbacks will be properly addressed to improve on its final output:

- The presence of thick edges at some locations can be addressed by non-maxima suppression.
- The problem of broken edges can be easily dealt with using edge linking techniques [2].

To reduce computational time, we can actually omit the optimization of the enhancement parameters (x_c, f_h). Furthermore, images that possess good contrast may not need contrast intensification at all.

7. Conclusions

In this paper, we have proposed a novel fuzzy-based parameterized Gaussian edge detector that utilizes both global (membership function based on gray level histogram) and local (membership function in a Gaussian

mask) properties in the fuzzy domain. In brief, our algorithm uniquely applies contrast intensification to enhance the image globally, and subsequently, a local Gaussian-type edge detector mask is used to find the edges. Both phases contain tunable parameters that can be adjusted through fuzzy entropy optimization to obtain more refined edges.

In our experiments on three classic test images, we have shown that this edge detector is immensely suitable for many applications such as face and fingerprint recognition as it does not distort the shape and is able to retain the important edges. For future directions, we intend to conduct a quantitative evaluation using known performance evaluation measures to make further comparisons with these popular techniques.

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